Stochastic and robust optimization algorithms for the inventory-routing problem

GHENT UNIVERSITY

FACULTY OF ENGINEERING AND ARCHITECTURE

Wouter Lefever

Wouter.Lefever@ugent.be

Supervisors: El-Houssaine Aghezzaf, Khaled Hadj-Hamou, Bernard Penz

Ghent University- Department of Industrial Systems Engineering and Product Design

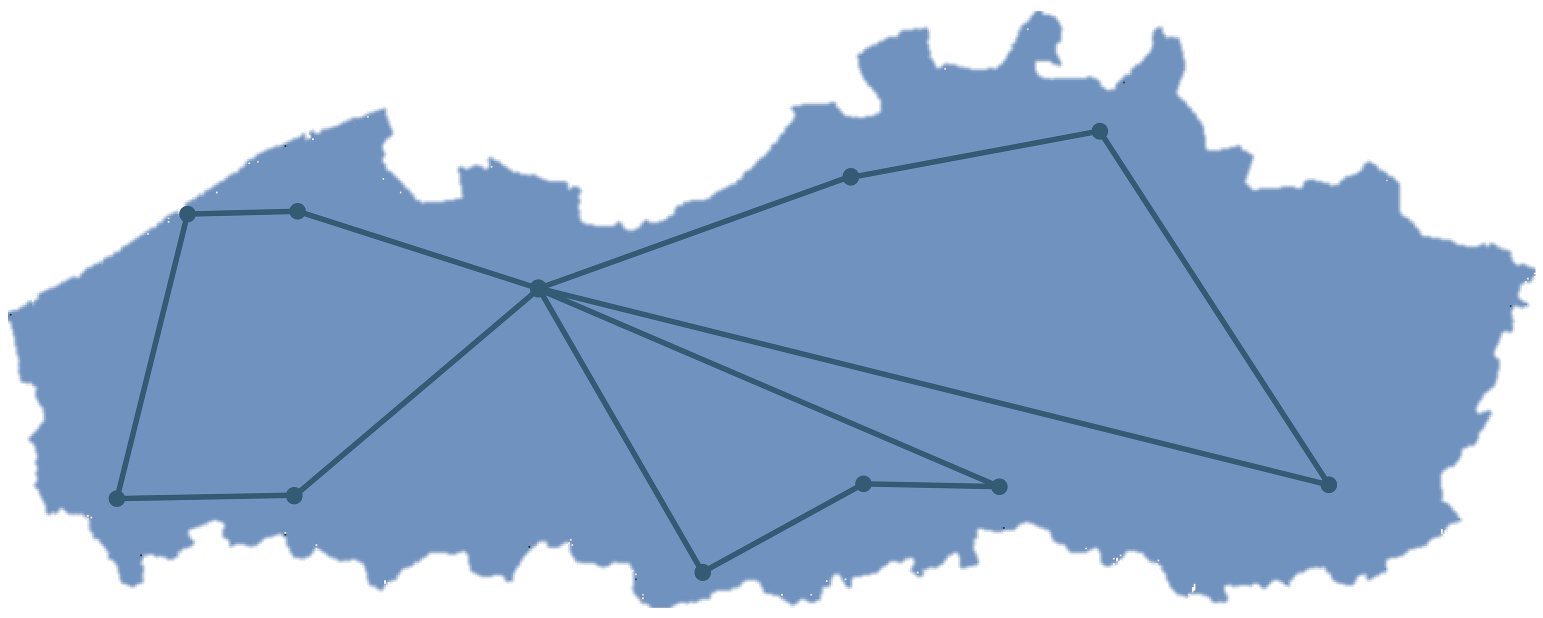
http://ea18.ugent.be

Introduction

The Inventory-Routing Problem (IRP) is a combined inventory management and routing problem that optimizes the distribution strategy over multiple periods. One of the main underlying assumptions in this problem is the invariability and predictability of exogenous factors such as travel times, demand, transportation costs, ...

The focus of this research is to develop optimization algorithms for the IRP under different conditions of a priori knowledge. We distinguish the following levels of knowledge on the uncertainty of the exogenous factors:

- No Information
- Basic Stochastic Information
- Full Stochastic Information
- Perfect Information



No Information

In the case that there is no information available on the uncertainty of the exogenous factors, the exogenous factors will be modeled as estimates. The problem can be solved as a deterministic optimization problem.

$$i \xrightarrow{t_{ij} = \hat{t}_{ij}(\xi) = 4h} j \longrightarrow \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} t_{ij} x_{ij} \leq T$$

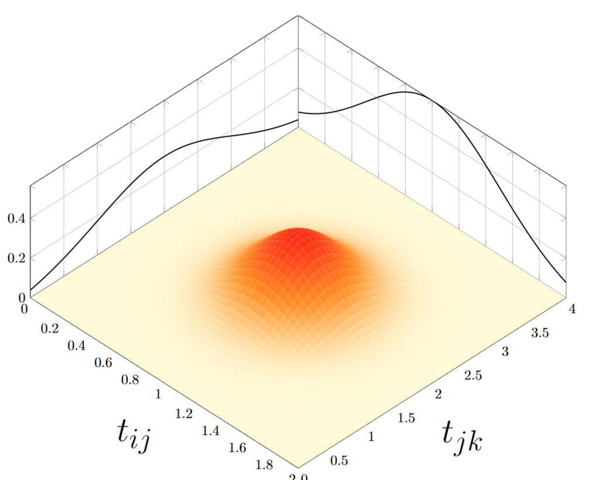
Full Stochastic Information

In the case of full stochastic information, we assume that the probability distribution of the exogenous factor is known. The problem can now be solved using stochastic programming methods. Typically, the problem is solved for a set of scenarios that reflect the underlying probability distribution.

$$i \xrightarrow{t_{ij} \sim \mathcal{N}(4h, 1h)} j \longrightarrow \forall s \in \mathcal{S} : \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} t_{ij}(\xi_s) x_{ij} \leq T$$

Challenge

It is important to choose the set of scenarios intelligently. The set should contain enough scenarios to approximate the probability distribution. On the other hand the complexity of the resulting stochastic program increases with the size of the scenario set. Variance reduction techniques such as antithetic variates may help to construct a set of limited size that represents the probability distribution effectively.



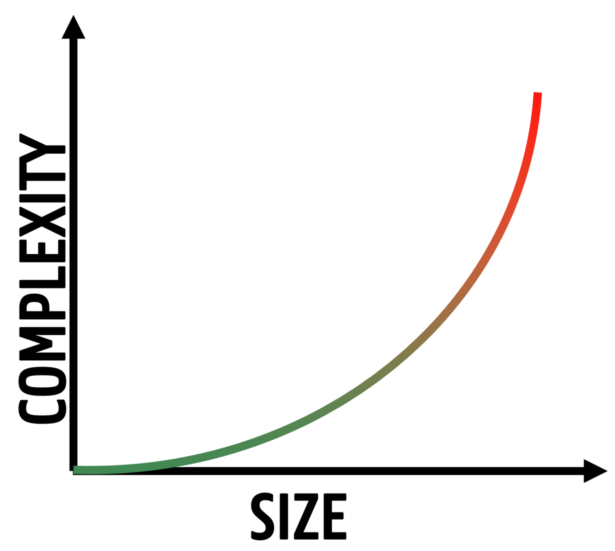
Basic Stochastic Information

In the case of basic stochastic information, we assume that the interval in which the exogenous factor takes values is known. The problem can no longer be solved as a deterministic optimization problem, but has to be reformulated. When the desired level of conservatism is determined, the robust counterpart can be solved such that the resulting solution will be protected against part of the uncertainty.

$$i \xrightarrow{t_{ij} \in [2h; 6h]} j \longrightarrow \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} t_{ij} x_{ij} + z\Gamma + \sum_{k \in \mathcal{J}} p_{ij}^{k} \leq T$$

Challenge

Determining the optimal solution of an IRP is an NP-complete problem. Using the current known algorithms the time required to solve the IRP increases very quickly as the size of the problem grows. The robust counterpart envelops the original deterministic problem. It is thus very important to pay attention to the computational complexity of the robust counterpart.



Perfect Information

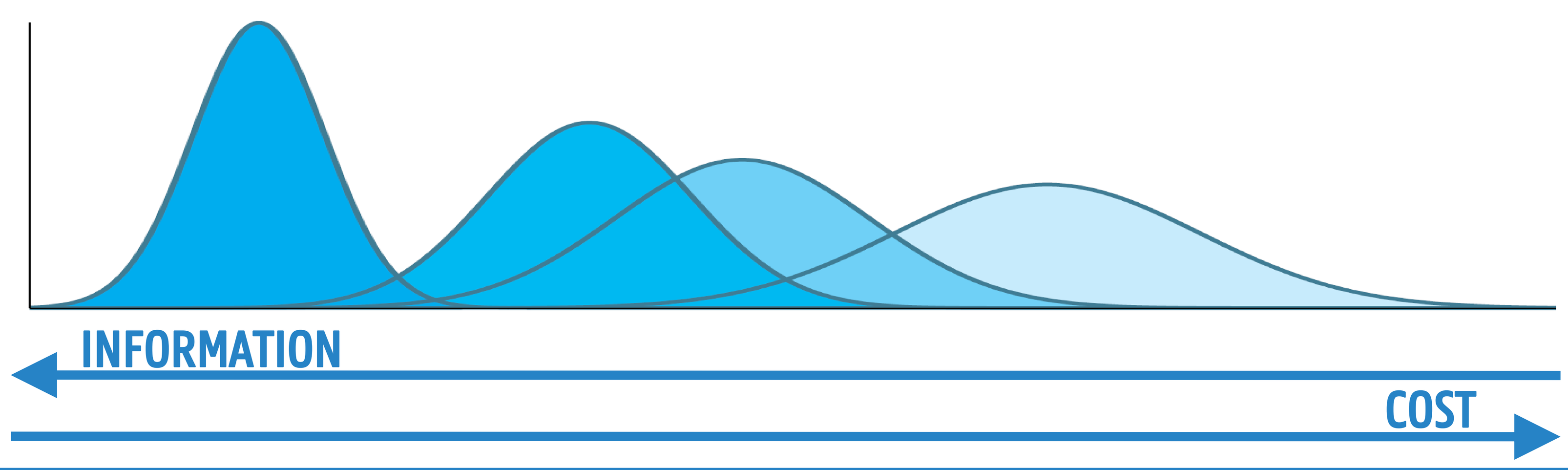
In the case of perfect information, the exact realization of the uncertainty is known a priori. However, in reality this is seldom the case. The problem can be solved as a deterministic optimization problem. The obtained solution will serve as a benchmark.

$$i \xrightarrow{t_{ij} = 4.26h} j \longrightarrow \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} t_{ij} x_{ij} \leq T$$

INFORMATION

Results

Using Monte Carlo simulation the cost of the solutions can be evaluated in a stochastic environment. The results show that the cost of the solution decreases according to the level of information available. The difference between two solutions indicates the profit that can be expected when more information is available.



Conclusion

In the Inventory-Routing Problem exogenous factors are generally assumed to be fixed and predictable, while in reality this is not the case. In this research we model the exogenous factors as uncertain parameters and distinguish four levels of knowledge on the uncertainty. We propose solution methods for the four levels of knowledge. The comparison of the cost of the solutions emphasizes:

1. The interest of modeling exogenous factors as uncertain parameters.
2. The gain that can be obtained by having additional information on the uncertainty of the exogenous factors.